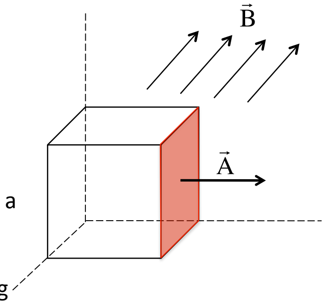
Problem 30.45

a.) We know that magnetic flux is defined as:

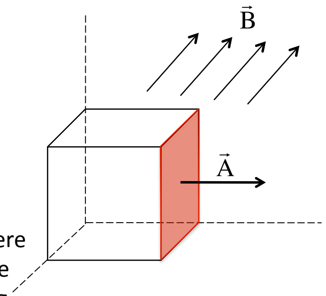
$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$



As the magnetic field is a defined, constant vector, and as the area vector simply has the magnitude of a side squared in the x-direction, this can also be written as $\vec{B}$ dotted into $\vec{A}$. That operation is executed in a unit vector notations by simply adding the like components multiplied together, or:

$$\begin{aligned}\Phi_B &= \vec{B} \bullet \vec{A} \\ &= \left(5\hat{i} + 4\hat{j} + 3\hat{k}\right)(T) \bullet \left(\left(2.5x10^{-2}\right)^2 \hat{i}\right)\left(m^2\right) \\ &= B_x A_x + B_y A_y + B_z A_z \\ &= (5\ T)\left(2.5x10^{-2}\ m\right)^2 + 0 + 0 \\ &= 3.12x10^{-3}\ T \bullet m^2 \\ &= 3.12x10^{-3}\ \text{Webers}\end{aligned}$$

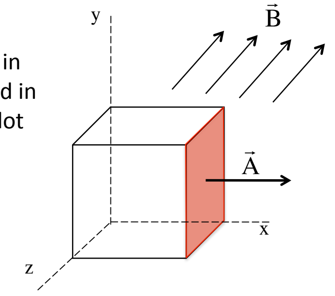
b.) ELECTRIC flux through a closed surface can be non-zero (hence, our ability to use Gauss's Law to derive electric-field functions) because there exist what are called *electric monopoles*—single electric charges (positive and/or negative) that can reside alone inside a Gaussian surface.



According to the Classical Theory of Magnetism, there can be no *magnetic monopoles*. That is to say, there will never be a south pole without an accompanying north pole. The consequence of this observation is that the flux lines *into* a closed surface will always equal the flux lines *out* of the surface. And the conclusion of *that* is that the *net magnetic flux* through any close surface will ALWAYS be zero.

So we're done with the problem and all is well, unless, of course, your name has the word NERD in parenthesis after it. Then you might be interested in doing more. Specifically, how might you do the dot product from a polar perspective (that is, using:

$$\Phi_m = \left|\vec{B}\right|\left|\vec{A}\right|\cos\theta$$



Where θ is the angle between $\vec{A}$ and $\vec{B}$.

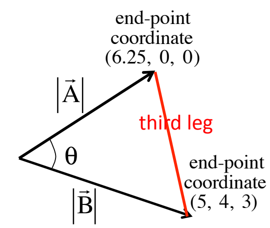
The magnitude of the area vector is:

$$\begin{aligned}\left|\vec{A}\right| &= L^2 \\ &= \left(2.5x10^{-2}\ m\right)^2 \\ &= 6.25x10^{-4}\ m^2\end{aligned}$$

The magnitude of the B-fld vector required a three dimensional version of the Pythagorean relationship, or :

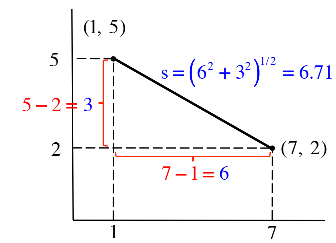
$$\begin{aligned}\left|\vec{B}\right| &= \left(B_x^2 + B_y^2 + B_z^2\right)^{1/2} \\ &= \left(5^2 + 4^2 + 3^2\right)^{1/2}(T) \\ &= 7.071067\ T\end{aligned}$$

The thrill comes in trying to determine the angle θ, or $\cos\theta$, between a vector in the $\hat{i}$ direction (that would be the area vector $\vec{A}$) and a vector in the "$5\hat{i} + 4\hat{j} + 3\hat{k}$" direction (that would be the magnetic field vector $\vec{B}$). This is not hard, but it's not altogether trivial. (I might add, the key to this was supplied by our very own, intrepid Mr. Fay).



If we could determine the third leg of the triangle shown above, we could use the Law of Cosines to determine what we need. So how do you determine the distance between two points in space? Consider the 2-d situation shown below.

To get the distance between the points (1,5) and (7,2), apparently we take the difference between the x coordinates (that yielded the "6" in our example), squared that number, do the same for the y coordinates (and the z coordinates if the problem is 3-d), add the squares, then take the square root. Easy!

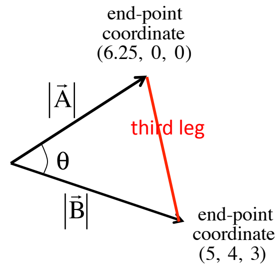


Trying our technique on our two vectors:

$$\vec{A} = 6.25\text{x}10^{-4}\,\hat{i}$$

and

$$\vec{B} = 5\hat{i} + 4\hat{j} + 3\hat{k}$$



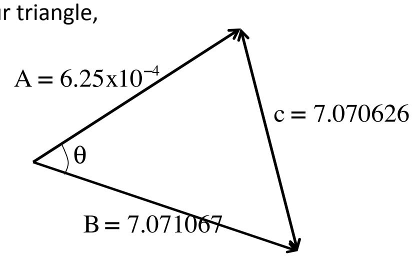
The endpoint for the first vector is at 6.25 along the x-axis. The endpoint for the second vector has an x-component of 5, a y-component of 4 and a z-component of 3. Using the "difference" approach we figured out on the previous page, we can determine the magnitude of the third vector as:

$$|\vec{s}| = \left[\left(A_x - B_x\right)^2 + \left(A_y - B_y\right)^2 + \left(A_z - B_z\right)^2\right]^{1/2}$$

$$= \left[\left(6.25\text{x}10^{-4} - 5\right)^2 + (-4)^2 + (-3)^2\right]^{1/2}$$

$$= 7.070626$$

We now have three vectors that make up our triangle, where the sides are as shown in the sketch.



So now THE LAW OF COSINES!

$$c^2 = A^2 + B^2 - 2AB\cos\theta$$

$$\Rightarrow \quad AB\cos\theta = \frac{-c^2 + A^2 + B^2}{2}$$

$$\Rightarrow \quad AB\cos\theta = \frac{-(7.070626)^2 + \left(6.25\text{x}10^{-4}\right)^2 + (7.071067)^2}{2}$$

$$= 3.12\text{x}10^{-3}$$

Great jumping huzzahs! Apparently:

$$\Phi_m = |\vec{B}||\vec{A}|\cos\theta$$

$$= 3.12\text{x}10^{-3}\ \text{W}$$

exactly matching the value we got using the saner approach. Dang we're good!